Homework #6
ENCE 302 - FALL 2001
Due M, 10/22

Problem 1:

Textbook: 3-32

Problem 3-32.

$$\lambda = 0.5,\ t = 1,\ \lambda t = 0.5,\ P(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} = \frac{0.5^x e^{-0.5}}{x!}$$

$$P(X = 2) = \frac{0.5^2 e^{-0.5}}{2!} = 0.0758$$

$$P(X = 0) = \frac{0.5^0 e^{-0.5}}{0!} = 0.6065$$

$$\lambda = 0.5,\ T = 20,\ \lambda T = 10,\ P(X = x) = \frac{(\lambda T)^x e^{-\lambda T}}{x!} = \frac{10^x e^{-10}}{x!}$$

$$P(1 \le X \le 2) = P(X = 1) + P(X = 2) = \frac{10^1 e^{-10}}{1!} + \frac{10^2 e^{-10}}{2!}$$

$$= 0.000454 + 0.00227 = 0.00272$$

Problem 2:

Textbook: 3-37

Problem 3-37.

$$\mu = 43'',\ \sigma = 6.5'',\ \Phi(z) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

(a) $$P(X > 55) = 1 - \Phi(z) = 1 - \Phi\left(\frac{55-43}{6.5}\right) = 1 - \Phi\left(\frac{12}{6.5}\right) \simeq 1 - \Phi(1.846)$$

$$= 1 - 0.967566 = 0.032434$$

(b) $$P(X < 35) = \Phi\left(\frac{35-43}{6.5}\right) = \Phi\left(\frac{-8}{6.5}\right) \simeq 1 - \Phi(-1.235) = 1 - \Phi(1.235)$$

$$= 1 - 0.890651 = 0.109349$$

(c) $$P(X < 32 \cup X > 51) = P(X < 32) + P(X > 51) = P(X < 32) + 1 - P(X \le 51)$$

$$= \Phi\left(\frac{32-43}{6.5}\right) + 1 - \Phi\left(\frac{51-43}{6.5}\right) = \Phi\left(\frac{-11}{6.5}\right) + 1 - \Phi\left(\frac{8.5}{6.5}\right)$$

$$\simeq \Phi(-1.6923) + 1 - \Phi(1.23) = 1 - \Phi(1.6923) + 1 - \Phi(1.23)$$

$$= 1 - 0.954486 + 1 - 0.890651 = 0.154836$$

(d) $$P(38 < X < 48) = P(X < 48) - P(X < 38) = \Phi\left(\frac{48-43}{6.5}\right) - \Phi\left(\frac{38-43}{6.5}\right)$$

$$= \quad \Phi\left(\frac{5}{6.5}\right) - \Phi\left(\frac{-5}{6.5}\right) = \Phi(0.77) - \Phi(-0.77) = \Phi(0.77) - [1 - \Phi(0.77)]$$

$$= \quad 0.77935 - 1 + 0.77935 = 0.5587$$

(e) Find x such that $P(X > x) = 0.05$, or

$0.05 = 1 - \Phi\left(\frac{x-43}{6.5}\right)$. Therefore, $x = 43 + (6.5)\Phi^{-1}(1 - 0.05) = 53.7''$

(f) Find x such that $P(X < x) = 0.01$, or

$0.01 = \Phi\left(\frac{x-43}{6.5}\right)$. Therefore, $x = 43 + (6.5)\Phi^{-1}(0.01) = 27.86''$

Problem 3:

Textbook: 3-39

Problem 3-39.

$$\mu_{R_e} = 2650, \quad \sigma_{R_e} = 700,$$

$$\sigma_Y^2 = \ln\left[1 + \left(\frac{\sigma_{R_e}}{\mu_{R_e}}\right)^2\right] = \ln\left[1 + \left(\frac{700}{2650}\right)^2\right] = 0.06745$$

$$\sigma_Y = 0.2597$$

$$\mu_Y = \ln(\mu_{R_e}) - \frac{1}{2}\sigma_Y^2 = \ln(2650) - \frac{1}{2}\sigma_Y^2 = 7.8486$$

$$P(2700 < R_e < 4000) = \Phi\left(\frac{\ln 4000 - 7.8486}{0.2597}\right) - \Phi\left(\frac{\ln 2700 - 7.8486}{0.2597}\right)$$

$$= \quad \Phi(1.72) - \Phi(0.20) = 0.957284 - 0.57926 = 0.378024$$

Problem 4:

Textbook: 3-41

# of hurricanes for all years = 4+2+1+3+0+2+…+ =30

$$\lambda = \frac{30}{15} = 2\ \frac{\text{times}}{\text{year}}$$

$$f(x) = \lambda e^{-\lambda t} = 2e^{-2t}$$

$$F_X(x) = 1 - e^{-2t}$$

$$\therefore P(X > 1) = 1 - F(1) = 0.1353$$

$$\text{Return Period} = \frac{1}{\lambda} = \frac{1}{2} = 0.5 \text{ year}$$