Homework #5 Solution
ENCE 302 - FALL 2001
Due W, 10/17

Problem 1:

Textbook: 3-12

Define: D = damaged item,
A = shipment by air,
S = shipment by sea,
G = shipment by ground

(a)

$$\begin{aligned}
\mathrm{P}(D) &= \mathrm{P}(D \cap A) + \mathrm{P}(D \cap G) + \mathrm{P}(D \cap S) \\
&= \mathrm{P}(D \mid A)\mathrm{P}(A) + \mathrm{P}(D \mid G)\mathrm{P}(G) + \mathrm{P}(D \mid S)\mathrm{P}(S) \\
&= 0.05 \times 0.2 + 0.1 \times 0.5 + 0.15 \times 0.3 \\
&= 0.105
\end{aligned}$$

(b)

$$\mathrm{P}(A \mid \mathrm{D}) = \frac{\mathrm{P}(D \cap A)}{\mathrm{P}(D)} = \frac{0.05 \times 0.2}{0.150} = \frac{0.010}{0.105} = 0.0952$$

$$\mathrm{P}(G \mid \mathrm{D}) = \frac{\mathrm{P}(D \cap G)}{\mathrm{P}(D)} = \frac{0.1 \times 0.5}{0.105} = \frac{0.050}{0.105} = 0.476$$

$$\mathrm{P}(S \mid \mathrm{D}) = \frac{\mathrm{P}(D \cap S)}{\mathrm{P}(D)} = \frac{0.15 \times 0.3}{0.105} = \frac{0.045}{0.105} = 0.429$$

Problem 2:

Textbook: 3-13

(a) Sample space of odd integers is $\{1,3,5\}$

Probability of getting three = $\frac{1}{3}$

(b) Sample space no face card = $\{$1 to 10 for each suit$\}$. There are 40 possible outcomes. The probability of getting a red ace = $\frac{1}{40}$

(c) Sample space = $\{$52 possible cards minus 5$\}$. There are 47 possible outcomes. The probability of getting an ace = $\frac{4-1}{47} = \frac{3}{47}$

Problem 3:

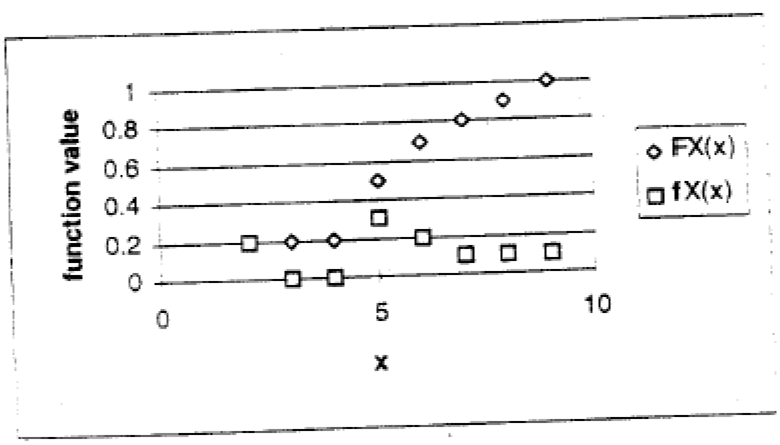
Textbook: 3-15

Problem 3-15

The PMF is

$$f_X(x) = \begin{cases} 0.2 & x = 2 \\ 0.0 & x = 3 \\ 0.0 & x = 4 \\ 0.3 & x = 5 \\ 0.2 \quad \text{for} & x = 6 \\ 0.1 & x = 7 \\ 0.1 & x = 8 \\ 0.1 & x = 9 \end{cases}$$

x	$F_X(x)$	$f_X(x)$
2	0.2	0.2
3	0.2	0
4	0.2	0
5	0.5	0.3
6	0.7	0.2
7	0.8	0.1
8	0.9	0.1
9	1	0.1



Problem 4:

Textbook: 3-17

Problem 3-17.

$$P(X = 4) = P(X \le 4) - P(X \le 3) = 0.2 - 0.2 = 0.$$

$$P(X = 5) = P(X \le 5) - P(X \le 4) = 0.5 - 0.2 = 0.3$$

$$P(X \le 5) = 0.5$$

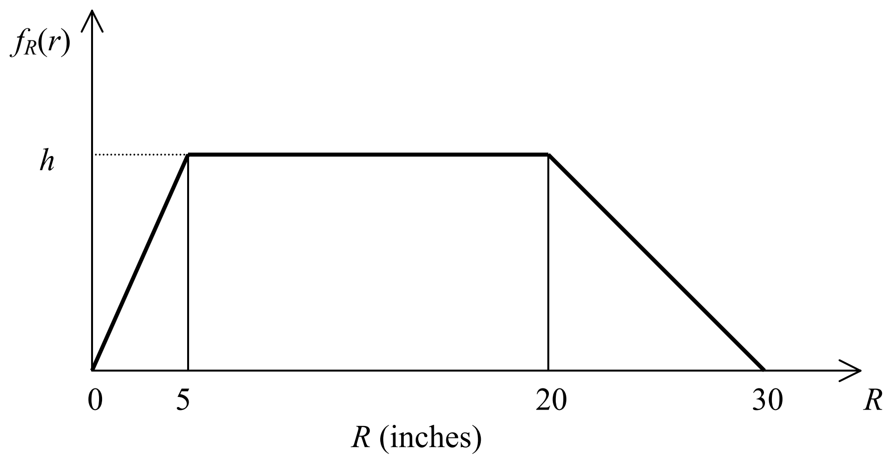
$$P(X < 5) = 0.2$$

$$P(4 < X \le 7) = P(X \le 7) - P(X \le 4) = 0.8 - 0.2 = 0.6$$

$$P(X \ge 7) = 1.0 - P(X < 7) = 1.0 - 0.7 = 0.3$$

Problem 5:

The PDF of the annual rainfall, *R*, of a city is provided in the following figure:



(a) Determine *h* that makes it a legitimate density function.
(b) Write a mathematical expression for its CDF, that is $F_R(r)$.
(c) Find the mean and the median values of *R*.
(d) Find the variance, standard deviation, and coefficient of variation (*COV*) of *R*.
(e) Determine the skewness of *R*.

***** SOLUTION *****

The area under the PDF must be 1.0. Thus, the height h of the PDF diagram sh
Figure P3.7 can be estimated as

$$\frac{1}{2}\times(5-0)\times h+(20-5)\times h+\frac{1}{2}\times(30-20)\times h=1.0$$

(a) or $\quad h=\dfrac{1}{22.5}$

Thus, the PDF of R can be defined as

$$\begin{aligned} f_R(r) &= \frac{1}{112.5}r, \quad 0 \le r \le 5 \\ &= \frac{1}{22.5}, \quad 5 \le r \le 20 \\ &= -\frac{1}{225}r+\frac{1}{7.5}, \quad 20 \le r \le 30 \\ &= 0, \quad \text{elsewhere.} \end{aligned}$$

$$E(R)=\mu_R=\int_{-\infty}^{\infty} r f_R(r)dr$$

$$=\int_0^5 r\left(\frac{1}{112.5}r\right)dr+\int_5^{20} r\left(\frac{1}{22.5}\right)dr+\int_{20}^{30} r\left(-\frac{1}{225}r+\frac{1}{7.5}\right)dr=13.889 \text{ inch}$$

Since mean is the centroidal distance of the PDF from the origin, it can also be calculated as:

$$E(R) = \left(\frac{5}{2}\times\frac{1}{22.5}\right)\times\left(\frac{2}{3}\times 5\right) + \left(15\times\frac{1}{22.5}\right)\times\left(5+\frac{15}{2}\right) + \left(\frac{10}{2}\times\frac{1}{22.5}\right)\times\left(20+\frac{10}{3}\right) = 13.889 \text{ inch}$$

The median value, r_m, is the value of R that satisfies $F_R(r_m) = 0.5$. We can calculate the CDF of R as

$$F_R(r) = \frac{1}{225}r^2, \quad 0 \le r \le 5$$

$$= \frac{1}{9} + \frac{1}{22.5}r, \quad 5 \le r \le 20$$

$$= \frac{7}{9} + -\frac{1}{450}r^2 + \frac{1}{7.5}r, \quad 20 \le r \le 30$$

Since $F_R(5) = \frac{1}{9}$ and $F_R(20) = \frac{7}{9}$, r_m should be in the range of 5 to 20.

Using the corresponding CDF, we can show that

$$F_R(r_m) = 0.5 \quad \text{or} \quad \frac{1}{9} + \frac{1}{22.5}r_m = 0.5$$

Median of $r = r_m = 13.75$ *inch*

The mode is the value of r with the highest PDF, i.e., $\frac{d}{dr}f_R(r) = 0$.

From Figure P3.7, it can be observed that the modal value is any value between 5 and 20.

Determine the variance, standard deviation, and coefficient of variation of R.

$$Variance = Var(R) = \int_{-\infty}^{\infty}(r-\mu_R)^2 f_R(r)dr$$

$$Var(R) = \int_0^5 (r-13.889)^2\left(\frac{1}{112.5}r\right)dr + \int_5^{20}(r-13.889)^2\left(\frac{1}{22.5}\right)dr$$

$$+ \int_{20}^{30}(r-13.889)^2\left(-\frac{1}{225}r + \frac{1}{7.5}\right)dr = 47.376 \text{ in.}^2$$

The variance can also be estimated by calculating the moment of inertia of the PDF about it mean as

$$Var(R) = \frac{1}{36}\left(\frac{1}{22.5}\right)\times 5^3 + \frac{1}{2}\left(\frac{1}{22.5}\right)\times 5\times\left(13.889-\frac{2}{3}\times 5\right)^2 + \frac{1}{3}\left(\frac{1}{22.5}\right)\times\left\{(13.889-5)^3 + (20-13.889)^3\right\} +$$

$$\frac{1}{36}\left(\frac{1}{22.5}\right)10^3 + \frac{1}{2}\left(\frac{1}{22.5}\right)\times 10\times\left(20-13.889+\frac{1}{3}\times 10\right)^2 = 13.786 + 33.590 = 47.376 \text{ in.}^2$$

The result is identical to the one obtained using Equation 3.14.

$$\sigma_R = \sqrt{Var(R)} = 6.883 \; in. \quad and \quad Cov(R) = \frac{6.883}{13.889} = 0.496$$

(f) Determine the skewness and skewness coefficient of R.

$$skewness = \int_{-\infty}^{\infty} (r - \mu_R)^3 f_R(r) dr$$

$$= \int_0^5 (r - 13.889)^3 \left(\frac{1}{112.5} r\right) dr + \int_5^{20} (r - 13.889)^3 \left(\frac{1}{22.5}\right) dr$$

$$+ \int_{20}^{30} (r - 13.889)^3 \left(-\frac{1}{225} r + \frac{1}{7.5}\right) dr = +34.278$$

$$skewness\ coefficient = \frac{34.278}{6.883^3} = 0.105$$

Problem 6:

Textbook: 3-28

Use the binomial distribution with the following parameter:

$p = 0.1$

(1) $\binom{10}{3} p^3 (1-p)^{10-3} = \frac{10!}{3!7!} (0.1)^3 (0.9)^7 = 0.0574$

(2) $\binom{20}{0} p^0 (1-p)^{20-0} = 0.9^{20} = 0.1216$

(3) $(1-p)^9 p^1 = (1-0.1)^9 (0.1) = 0.0387$

(4) $p^3 = 0.1^3 = 0.001$